Bifurcation Control of Nonlinear Systems

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Bifurcation Control of Nonlinear Systems†

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Abstract

Bifurcation control is discussed in the context of the stabilization of high angle-of-attack flight dynamics. Two classes of stabilization problems for which bifurcation control is useful are discussed. In the first class, which is emphasized in this presentation, a nonlinear control system operates at an equilibrium point which persists only under very small perturbations of a parameter. Such a system will tend to exhibit a jump, or divergence, instability in the absence of appropriate control action. In the second class of systems, an instance of which arises in a tethered satellite system model [14], eigenvalues of the system linearization appear on (or near) the imaginary axis in the complex plane, regardless of the values of system parameters or admissible linear feedback gains.

1. Introduction

The important role played by concepts from bifurcation theory in the sciences, engineering and the social sciences is well-established (e.g., [7], [12], [15], [19]). Nonlinear phenomena such as the appearance of limit cycles, divergence to new steady states, and transition to chaotic behavior have been observed and studied

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for a great variety of systems. Only recently have issues of the control of such nonlinear phenomena been given serious consideration (e.g., [1]-[2], [4], [9], [14], [16], [17]). Thus, the theory of control of bifurcations, as well as that of controlling chaos, is in its infancy. In this note, some results of a program of research in bifurcation control and applications are presented. Emphasis is placed on concepts and on the motivation provided by applications. Explicit calculations and other technical details can be found in references [1]-[4], [10], [14].

The paper is organized as follows. In the next section, some of the main questions considered in bifurcation control are discussed along with representative results. In Section 3, we consider control law design for an aircraft model at high angle-of-attack. Concluding remarks appear in Section 4.

2. Bifurcation Control Framework

Consider a nonlinear system

$$\dot{x} = F_{\mu}(x) \tag{1}$$

where $x \in I\!R^n$, μ is a real parameter and F is sufficiently smooth. Suppose (1) has an equilibrium point $x_0(\mu)$ which exists and is asymptotically stable for a range of parameter values. Outside the normal operating regime, i.e., as μ is varied, the operating point can lose its stability in a number of ways. For instance, a complex conjugate pair of eigenvalues of the linearization of (1) at x_0 may cross the imaginary axis into the right half of the complex plane as μ is varied through a critical value μ_c. Alternately, the equilibrium point might cease to exist past a parameter value for which the linearization has a zero eigenvalue. In the first of these simple routes to instability, the system (1) is known to undergo a *Hopf bifurcation* to periodic solutions. In the second, a *fold* or *saddle-node bifurcation* occurs.

Generically, the situation in the case of a Hopf bifurcation can be further classified according to whether the bifurcation is subcritical or supercritical. To describe these possibilities further, denote by μ_c the critical parameter value, and suppose that $x_0(\mu)$ is stable for $\mu < \mu_c$ but unstable for $\mu > \mu_c$. Fig. (1a) illustrates the subcritical Hopf bifurcation, wherein unstable periodic orbits of small amplitude emerge from $x_0(\mu_c)$ and exist, locally, for $\mu < \mu_c$. In the supercritical Hopf bifurcation, a stable periodic orbit emerges at μ_c, and exists for $\mu > \mu_c$ (see Fig. 1(b)).

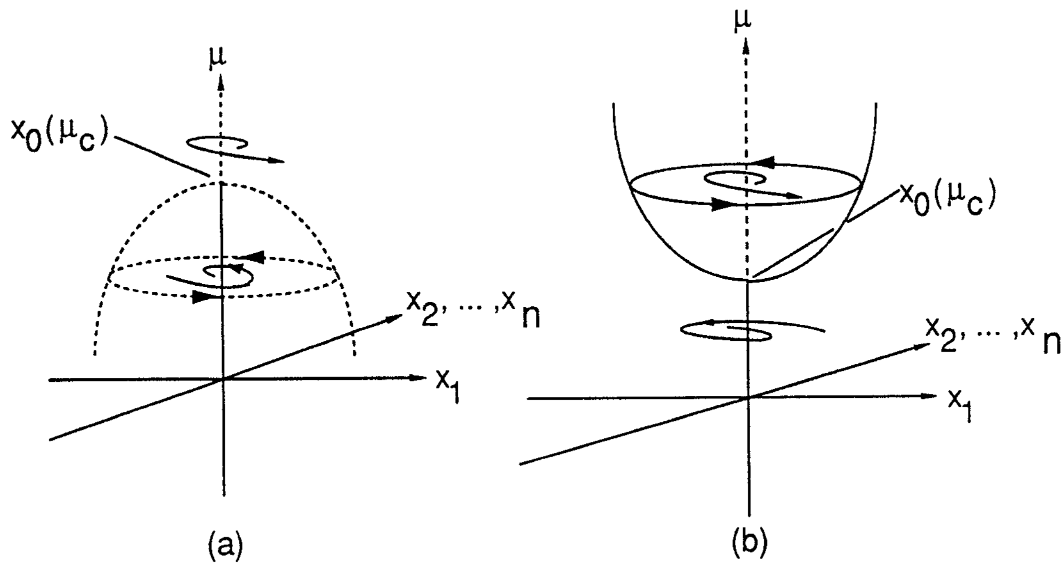

Fig. 1. (a) Subcritical, and (b) Supercritical Hopf Bifurcation

From Fig. 1(a), in the subcritical case an initial condition near $x_0(\mu)$ for $\mu > \mu_c$ will tend to diverge away from the nominal equilibrium. In contrast, as seen from Fig. 1(b), for a supercritical Hopf bifurcation the same initial condition would result in an oscillatory motion in the immediate vicinity of x_0. Thus, the supercritical bifurcation results in a more desirable system response than the subcritical bifurcation, locally near $\mu = \mu_c$. This observation, when considered along with other related local and global issues [1], [3], [4], [9], [10], leads to questions of stabilizability of Hopf bifurcations.

In [1], the stabilization of Hopf bifurcations by smooth feedback is considered. Specifically, local bifurcation control deals with the design of smooth control laws $u = u(x)$ which stabilize a bifurcation occurring in a one-parameter family of systems

$$\dot{x} = f_\mu(x, u). \tag{2}$$

These control laws exist generically, even if the critical eigenvalues of the linearized system at the equilibrium of interest are uncontrollable. It is shown that rendering the bifurcation supercritical also achieves asymptotic stability of the equilibrium x_0 at criticality. Explicit computations yielding stabilizing control laws are also given in [1].

This approach has been employed in the design of stabilizing control laws for a tethered satellite system in the station-keeping mode [14]. In this application, a pair of *uncontrollable* pure imaginary eigenvalues appears in the system lineariza-

tion. In [14], local bifurcation control is employed to stabilize the equilibrium using linear and nonlinear tether tension control laws.

3. An Application to High α Flight Control

Several authors have studied the nonlinear phenomena that arise commonly in aircraft flight at high angle-of-attack (alpha). The literature on high alpha flight dynamics, control and aerodynamics has grown at a rapid pace. Of particular relevance here are references [6], [8], [11], and [16]. The direct linkage of aircraft stall and divergence, as well as other nonlinear aircraft motions in high incidence flight, to bifurcations of the governing dynamic equations is a goal of many previous investigations.

In [4], we study the stabilization of the trim condition of an aircraft arbitrarily close to the stall angle, in a manner which also provides an *impending stall warning signal* to the pilot. This signal is a small-amplitude, stable limit cycle-type pitching motion of the aircraft which persists to within a prescribed margin from impending divergent stall. This is a Hopf-bifurcated periodic solution of the system dynamics, which is stabilized using the methods of bifurcation control. From [11, Eqs. (10), (11)], we have the following model for pitching motions of a model F-8 Crusader aircraft in nearly level flight (i.e., for pitch angle remaining small). Here, α = angle-of-attack, θ = pitch angle, $\dot{\theta}$ = pitching moment, and δ_e = the instantaneous elevator control surface deflection.

$$\begin{aligned} \dot{\alpha} =& \dot{\theta} - \alpha^2\dot{\theta} - 0.088\alpha\dot{\theta} - 0.877\alpha + 0.47\alpha^2 + 3.846\alpha^3 \\ & - 0.215\delta_e + 0.28\delta_e\alpha^2 + 0.47\delta_e^2\alpha + 0.63\delta_e^3 \end{aligned} \tag{3a}$$

$$\begin{aligned} \ddot{\theta} =& - 0.396\dot{\theta} - 4.208\alpha - 0.47\alpha^2 - 3.564\alpha^3 \\ & - 20.967\delta_e + 6.265\delta_e\alpha^2 + 46\delta_e^2 + 61.4\delta_e^3 \end{aligned} \tag{3b}$$

We have studied the stability of this model as a function of δ_e viewed as a *parameter*, as well as stabilization of the trim condition using elevator deflection as a feedback control signal which can either be linear or nonlinear. In either case, we seek control laws which have a negligible effect on the trim condition, which itself depends on δ_e. To achieve this, we require a certain form of dependence of the control signal on the state, namely

$$\begin{aligned} \delta_e(x) = \delta_{eC} + \{ & \text{ a polynomial in } (x_1 - x_{10}(\delta_{eC})) \\ & \text{and } (x_2 - x_{20}(\delta_{eC}))\}. \end{aligned} \tag{4}$$

Here, x_1 and x_2 are the state variables α and $\dot{\theta}$, respectively, δ_{eC} is the constant *commanded* value of δ_e, and subscripts 0 indicate equilibrium (trim) values of state variables, which depend on δ_{eC}. In our example, curve fitting gives an approximation for the trim condition as a function of δ_{eC} [4].

The design procedure aims to result in an increased range of stable angles-of-attack. First, a linear feedback complying with the general form (4) is designed to stabilize the trim condition for all values of δ_{eC} up to a value which verges on stall. Next, a nonlinear controller is designed to control the stability of the bifurcation which occurs at the point of instability just prior to stall. This bifurcation is a Hopf bifurcation to periodic solutions. By ensuring a small amplitude stable periodic solution in the neighborhood of the unstable trim condition, a signal of incipient stall is produced (a stall warning signal). This is achieved through the addition of nonlinear (quadratic and cubic) terms to the linear feedback, as follows:

$$\begin{aligned}\delta_e =& \delta_{eC} + k_1(\alpha - \alpha_0(\delta_{eC})) + k_2(\dot{\theta} - \dot{\theta}_0(\delta_{eC})) \\ &+ q_1(\alpha - \alpha_0(\delta_{eC}))^2 + h_1(\alpha - \alpha_0(\delta_{eC}))^3 \\ &+ h_2(\dot{\theta} - \dot{\theta}_0(\delta_{eC}))^3 \end{aligned} \tag{5}$$

Here, $q_1 = h_1 = h_2 = 0.8$, resulting in a supercritical Hopf bifurcation (as seen by applying the software tool BIFOR2 [12]). Fig. 2 illustrates the conclusions.

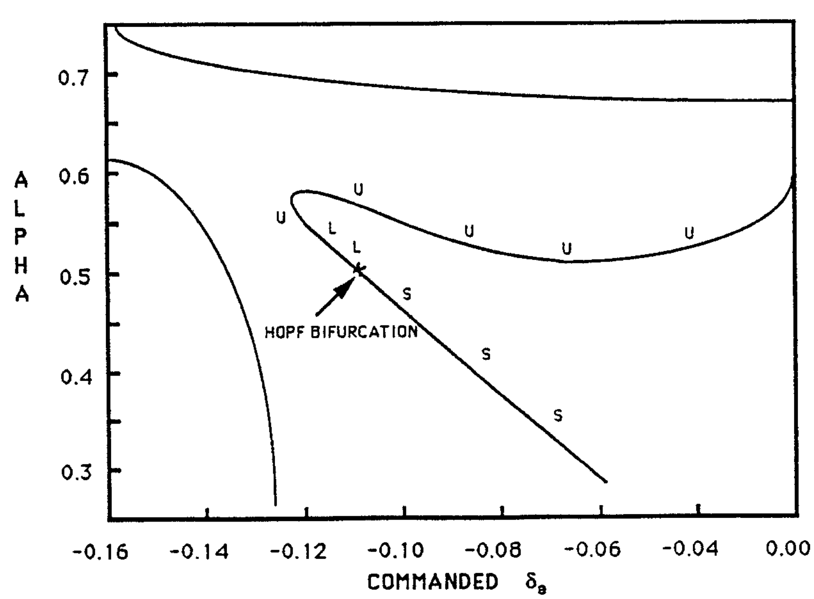

Fig 2. Bifurcation diagram for controlled model of F-8

As shown in [4], this results in significantly extending the operating envelope. The stable limit cycle ("L" in Fig. 2) bifurcates via a homoclinic orbit and then vanishes. System trajectories near the nominal equilibrium for parameter values

past this "homoclinic" value but prior to the "fold" critical parameter value, will diverge, no longer converging to a stable limit cycle.

4. Discussion

Only certain fundamental aspects of bifurcation control problems and applications were discussed in this note. We mention several problems which are currently under investigation. The suboptimal design of stabilizing controllers for nonlinear systems bordering on instability is being considered [10]. Applications of bifurcation control ideas in areas such as active stall mitigation in jet engines [3] and control of voltage collapse in electric power systems are also being addressed. Finally, extensions are suggested by interesting recent work on the control of chaos [17].

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